Published by Institute of Physics Publishing for SISSA
Received: *February 18, 2008*
Revised: *March 27, 2008*
Accepted: *April 7, 2008*
Published: *April 18, 2008*

Group space scan of flavor symmetries for nearly tribimaximal lepton mixing

Florian Plentinger, Gerhart Seidl and Walter Winter

Institut für Theoretische Physik und Astrophysik Universität Würzburg, D-97074 Würzburg, Germany
E-mail: fplentinger@physik.uni-wuerzburg.de, seidl@physik.uni-wuerzburg.de, winter@physik.uni-wuerzburg.de

Abstract: We present a systematic group space scan of discrete Abelian flavor symmetries for lepton mass models that produce nearly tribimaximal lepton mixing. In our models, small neutrino masses are generated by the type-I seesaw mechanism. The lepton mass matrices emerge from higher-dimension operators via the Froggatt-Nielsen mechanism and are predicted as powers of a single expansion parameter ϵ that is of the order of the Cabibbo angle $\theta_{\mathrm{C}} \simeq 0.2$. We focus on solutions that can give close to tribimaximal lepton mixing with a very small reactor angle $\theta_{13} \approx 0$ and find several thousand explicit such models that provide an excellent fit to current neutrino data. The models are rather general in the sense that large leptonic mixings can come from the charged leptons and/or neutrinos. Moreover, in the neutrino sector, both left- and right-handed neutrinos can mix maximally. We also find a new relation $\theta_{13} \lesssim \mathcal{O}(\epsilon^3)$ for the reactor angle and a new sum rule $\theta_{23} \approx \frac{\pi}{4} + \epsilon/\sqrt{2}$ for the atmospheric angle, allowing the models to be tested in future neutrino oscillation experiments.

Keywords: Discrete and Finite Symmetries, Solar and Atmospheric Neutrinos, Neutrino Physics.

Contents

1. Introduction

During the past decade, solar [1], atmospheric [2], reactor [3], and accelerator [4] neutrino oscillation experiments, have very well established that neutrinos are massive. Since neutrinos are massless in the standard model (SM), massive neutrinos signal physics beyond the SM. In fact, the smallness of neutrino masses $\sim 10^{-2} \ldots 10^{-1}$ eV can be naturally connected with grand unified theories (GUTs) [5, 6] via the seesaw mechanism [7, 8], in which the absolute neutrino mass scale becomes suppressed by an energy scale close to the GUT scale $M_{\mathrm{GUT}} \approx 2 \times 10^{16}$GeV [9].

Current neutrino oscillation data (for a recent global fit see ref. [10]) tells us that the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) lepton mixing matrix U_{PMNS} [11] can be approximated by the Harrison-Perkins-Scott (HPS) tribimaximal mixing matrix U_{HPS} [12] as

$$U_{\mathrm{PMNS}} \approx U_{\mathrm{HPS}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{1.1}$$

In U_{HPS}, the solar angle θ_{12} and the atmospheric angle θ_{23} are given by $\theta_{12} = \arctan(1/\sqrt{2})$ and $\theta_{23} = \pi/4$, whereas the reactor angle θ_{13} vanishes, i.e., $\theta_{13} = 0$. The actually observed leptonic mixing angles in U_{PMNS} may then be expressed in terms of deviations from tribimaximal mixing [13, 14] as "nearly" or "near" tribimaximal lepton mixing [15].

Many models have been proposed in the literature to reproduce tribimaximal leptonic mixing using non-Abelian flavor symmetries (for early models based on A_4 and examples using the double covering group of A_4, see refs. [16] and [17]). These models, however, have generally difficulties (for a discussion see ref. [18]) to predict the observed fermion mass hierarchies and the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix $V_{\rm CKM}$ [19] (see ref. [20] for models including quarks and ref. [21] for unified models). An interesting connection between the quark and the lepton sector, on the other hand, is implied by the idea of quark-lepton complementarity (QLC) [22], which is the phenomenological observation that the measured solar mixing angle very accurately satisfies the relation $\theta_{12} + \theta_{\rm C} \approx \pi/4$, where $\theta_{\rm C} \simeq 0.2$ is the Cabibbo angle. QLC has been studied from many different points of view: as a correction to bimaximal mixing [23], together with sum rules [24], with stress on phenomenological aspects [25], in conjunction with parameterizations of $U_{\rm PMNS}$ in terms of $\theta_{\rm C}$ [26], with respect to statistical arguments [27], by including renormalization group effects [28], and in model building realizations [29].

In refs. [30, 31], we have suggested a generalization of QLC to "extended QLC" (EQLC), where the mixing angles of the left- and right-handed leptons can assume any of the values $\frac{\pi}{4}, \epsilon, \epsilon^2, 0$. Here, ϵ is of the order of the Cabibbo angle $\epsilon \simeq 0.2$. By expressing also the lepton mass ratios as powers of ϵ, we have derived in ref. [31] for the CP-conserving case (a discussion of nonzero phases can be found in ref. [32]) in total 1981 qualitatively distinct mass matrix textures for the charged leptons and neutrinos that lead to nearly tribimaximal neutrino mixing with a small reactor angle $\theta_{13} \approx 0$. For these textures, the neutrino masses become small due to the canonical type-I seesaw mechanism (for a related approach see ref. [33]). The matrix elements of these textures are in the flavor basis all expressed by powers of ϵ, which serves as a single small expansion parameter of the matrices. This suggests a model building interpretation of the textures in terms of flavor symmetries, e.g. via the Froggatt-Nielsen mechanism [34].

In this paper, we describe the systematic construction of several thousand explicit lepton mass models, in which nearly tribimaximal lepton mixing and the mass hierarchies of charged leptons and neutrinos emerge from products of discrete Abelian flavor symmetries. Our motivation is that models with Abelian flavor symmetries generally have the merit that they need only a very simple scalar sector to achieve the necessary flavor symmetry breaking. We perform a group space scan of products of discrete Z_n flavor symmetries using the results of EQLC from ref. [31] and restrict to the case of real lepton mass matrices, i.e., we consider the CP-conserving case. In order to generate small neutrino masses, we assume only the canonical type-I seesaw mechanism. The hierarchical pattern of the lepton mass matrices results from higher-dimension operators that are produced by the Froggatt-Nielsen mechanism. Moreover, we are interested only in flavor symmetries that yield nearly tribimaximal lepton mixing with a very small reactor angle $\theta_{13} \approx 0$.

The paper is organized as follows: in section 2, we introduce the notation for the lepton masses and mixings. Next, in section 3, we specify our discrete flavor symmetries and describe the generation of lepton mass terms via the Froggatt-Nielsen mechanism. Then, we outline in section 4 our approach to the group space scan of flavor symmetries. Our general results, a list of explicit example models, a new relation for the reactor angle, and our sum rules for the PMNS angles including a new sum rule for the atmospheric angle, are shown in section 5. Finally, we present in section 6 our summary and conclusions.

JHEP04(2008)077

2. Lepton masses and mixings

We assume the SM with gauge group $G_{\rm SM} = {\rm SU}(3)_c \times {\rm SU}(2)_L \times {\rm U}(1)_Y$ plus three right-handed neutrinos that generate small neutrino masses via the type-I seesaw mechanism [7]. The lepton Yukawa couplings and mass terms are

$$\mathcal{L}_{\rm Y} = -(Y_\ell)_{ij} H^* \ell_i e_j^c - (Y_D)_{ij} {\rm i}\sigma^2 H \ell_i \nu_j^c - \frac{1}{2}(M_R)_{ij} \nu_i^c \nu_j^c + {\rm h.c.}, \tag{2.1}$$

where $\ell_i = (\nu_i, e_i)^T$, e_i^c, and ν_i^c, are the left-handed leptons, the right-handed charged lepton doublets, and the right-handed SM singlet neutrinos, and $i = 1, 2, 3$ is the generation index. Here, H is the SM Higgs doublet, Y_ℓ and Y_D are the Dirac Yukawa coupling matrices of the charged leptons and neutrinos, and M_R is the Majorana mass matrix of the right-handed neutrinos with entries of the order of the $B - L$ breaking scale $M_{B-L} \sim 10^{14}$ GeV. After electroweak symmetry breaking, H develops a vacuum expectation value $\langle H \rangle \sim 10^2$ GeV, and the mass terms of the leptons become

$$\mathcal{L}_{\rm mass} = -(M_\ell)_{ij} e_i e_j^c - (M_D)_{ij} \nu_i \nu_j^c - \frac{1}{2}(M_R)_{ij} \nu_i^c \nu_j^c + {\rm h.c.}, \tag{2.2}$$

where $M_\ell = \langle H \rangle Y_\ell$ is the charged lepton and $M_D = \langle H \rangle Y_D \sim 10^2$ GeV the Dirac neutrino mass matrix. After integrating out the right-handed neutrinos, the seesaw mechanism leads to the effective Majorana neutrino mass matrix

$$M_{\rm eff} = -M_D M_R^{-1} M_D^T\,, \tag{2.3}$$

with entries of the order 10^{-2} eV in agreement with observation. The leptonic Dirac mass matrices M_ℓ and M_D, and the Majorana mass matrices M_R and $M_{\rm eff}$ are diagonalized by

$$M_\ell = U_\ell M_\ell^{\rm diag} U_{\ell'}^\dagger, \quad M_D = U_D M_D^{\rm diag} U_{D'}^\dagger, \quad M_R = U_R M_R^{\rm diag} U_R^T, \quad M_{\rm eff} = U_\nu M_{\rm eff}^{\rm diag} U_\nu^T\,, \tag{2.4}$$

where $U_\ell, U_{\ell'}, U_D, U_{D'}, U_R$, and U_ν, are unitary mixing matrices, whereas $M_\ell^{\rm diag}, M_D^{\rm diag}$, $M_R^{\rm diag}$, and $M_{\rm eff}^{\rm diag}$, are diagonal mass matrices with positive entries. The mass eigenvalues of the charged leptons and neutrinos are given by $M_\ell^{\rm diag} = {\rm diag}(m_e, m_\mu, m_\tau)$ and $M_{\rm eff}^{\rm diag} = {\rm diag}(m_1, m_2, m_3)$, where m_1, m_2, and m_3, are the first, second, and third neutrino mass eigenvalues. We can always write a mixing matrix U_x as a product of the form

$$U_x = D_x \widehat{U}_x K_x, \tag{2.5}$$

where $\widehat{U}_x$ is a CKM-like matrix that reads in the standard parameterization (we follow here throughout the conventions and definitions given in ref. [30])

$$\widehat{U}_x = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-{\rm i}\widehat{\delta}^x} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{{\rm i}\widehat{\delta}^x} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{{\rm i}\widehat{\delta}^x} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{{\rm i}\widehat{\delta}^x} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{{\rm i}\widehat{\delta}^x} & c_{23}c_{13} \end{pmatrix}, \tag{2.6}$$

JHEP04(2008)077

with $s_{ij} = \sin\hat{\theta}^x_{ij}$, $c_{ij} = \cos\hat{\theta}^x_{ij}$, and $\hat{\theta}^x_{ij} \in \{\hat{\theta}^x_{12}, \hat{\theta}^x_{13}, \hat{\theta}^x_{23}\}$ lie all in the first quadrant, i.e., $\hat{\theta}_{ij} \in [0, \frac{\pi}{2}]$, and $\widehat{\delta}^x \in [0, 2\pi]$. In eq. (2.5), D_x and K_x denote diagonal phase matrices that are $D_x = \mathrm{diag}(e^{\mathrm{i}\varphi^x_1}, e^{\mathrm{i}\varphi^x_2}, e^{\mathrm{i}\varphi^x_3})$ and $K_x = \mathrm{diag}(e^{\mathrm{i}\alpha^x_1}, e^{\mathrm{i}\alpha^x_2}, 1)$, where the index x runs over $x = \ell, \ell', D, D', R, \nu$. The phases in D_x and K_x are all in the range $[0, 2\pi]$. The PMNS matrix reads

$$U_{\mathrm{PMNS}} = U^\dagger_\ell U_\nu = \widehat{U}_{\mathrm{PMNS}} K_{\mathrm{Maj}}, \tag{2.7}$$

where $\widehat{U}$ is a CKM-like matrix parameterized as in eq. (2.6), and $K_{\mathrm{Maj}} = \mathrm{diag}(e^{\mathrm{i}\phi_1}, e^{\mathrm{i}\phi_2}, 1)$ contains the Majorana phases ϕ_1 and ϕ_2. The CKM-like matrix $\widehat{U}_{\mathrm{PMNS}}$ in eq. (2.7) is described by the solar angle θ_{12}, the reactor angle θ_{13}, the atmospheric angle θ_{23}, and the Dirac CP-phase δ, which we identify in the parameterization of eq. (2.6) as $\hat{\theta}^x_{ij} \to \theta_{ij}$ and $\widehat{\delta}^x \to \delta$.

3. Flavor structure from Z_n symmetries

Let us next extend the SM gauge group to $G_{\mathrm{SM}} \times G_F$, where G_F is a flavor symmetry. We assume that G_F is a direct product of discrete Z_n symmetries, i.e.,

$$G_F = Z_{n_1} \times Z_{n_2} \times \cdots \times Z_{n_m}, \tag{3.1}$$

where m is the number of Z_n factors and the n_k ($k = 1, 2, \ldots, m$) may be different. We will denote by $|G_F| = \Pi^m_{k=1} n_k$ the group order (i.e., the number of elements) of G_F. Under G_F, we assign to the leptons the charges

$$e^c_i \sim (p^i_1, p^i_2, \ldots, p^i_m) = p^i, \quad \ell_i \sim (q^i_1, q^i_2, \ldots, q^i_m) = q^i, \quad \nu^c_i \sim (r^i_1, r^i_2, \ldots, r^i_m) = r^i, \tag{3.2}$$

where the jth entry in each row vector denotes the Z_{n_j} charge of the particle and $i = 1, 2, 3$ is the generation index (see section 2). In the following, we choose a convention, where for each group Z_{n_k} the charges are non-negative and lie in the range

$$p^i_k, q^i_k, r^i_k \in \{0, 1, 2, \ldots, n_k - 1\}. \tag{3.3}$$

The sum $x_k + y_k$ of two Z_{n_k} charges is equivalent with $x_k + y_k \bmod n_k$ and only determined modulo n_k. In order to spontaneously break the flavor symmetries, we assume for each factor Z_{n_k} a single scalar flavon field f_{n_k} that carries a charge $-1 \sim n_k - 1$ under Z_{n_k} but is a singlet under all other Z_{n_j} with $j \neq k$. Moreover, the f_{n_k} are G_{SM} singlets.

When the f_{n_k} acquire nonzero universal vacuum expectation values $\langle f_{n_k} \rangle \simeq v$, non-renormalizable lepton Yukawa couplings and mass terms of the form

$$\begin{aligned}\mathcal{L}_{\mathrm{Y}} = &-(\Pi^m_{k=1}\epsilon^{a^k_{ij}})(Y'_\ell)_{ij} H^* \ell_i e^c_j - (\Pi^m_{k=1}\epsilon^{b^k_{ij}})(Y'_D)_{ij} \mathrm{i}\sigma^2 H \ell_i \nu^c_j \\ &-\frac{1}{2}(\Pi^m_{k=1}\epsilon^{c^k_{ij}}) M_{B-L} (Y'_R)_{ij} \nu^c_i \nu^c_j + \mathrm{h.c.}\end{aligned} \tag{3.4}$$

are generated by the Froggatt-Nielsen mechanism by integrating out heavy fermions with universal mass $\simeq M_F$, where $\epsilon = v/M_F \simeq \theta_{\mathrm{C}} \simeq 0.2$ is of the order of the Cabibbo angle, and

$$\begin{aligned} a_{ij}^k &= \min\{p_i^k + q_j^k \bmod n_k, -p_i^k - q_j^k \bmod n_k\}, \\ b_{ij}^k &= \min\{q_i^k + r_j^k \bmod n_k, -q_i^k - r_j^k \bmod n_k\}, \\ c_{ij}^k &= \min\{r_i^k + r_j^k \bmod n_k, -r_i^k - r_j^k \bmod n_k\}, \end{aligned} \tag{3.5}$$

whereas Y'_ℓ, Y'_D, and Y'_R, are dimensionless order unity Yukawa couplings. The modulo function $\bmod n_k$ in eq. (3.5) is a consequence of the cyclic nature of the Z_n symmetries, and the minimum takes into account that the higher dimension operators can be built from both f_{n_k} and the complex conjugated fields $f_{n_k}^*$. The important point is that the Z_{n_k} charges of the leptons determine a hierarchical pattern of the Yukawa coupling matrices and the right-handed Majorana neutrino mass matrix.

We define a "texture" as the matrix collecting the leading order products of ϵ for a certain Yukawa coupling or mass matrix, thereby ignoring the information on the order unity coefficients Y'_ℓ, Y'_D, and Y'_R. The lepton textures are therefore the 3×3 matrices with matrix elements approximating $(M_\ell)_{ij}, (M_D)_{ij}$, and $(M_R)_{ij}$, as

$$(M_\ell)_{ij} \approx \Pi_{k=1}^m \epsilon^{a_{ij}^k}, \quad (M_D)_{ij} \approx \Pi_{k=1}^m \epsilon^{b_{ij}^k}, \quad (M_R)_{ij} \approx \Pi_{k=1}^m \epsilon^{c_{ij}^k}. \tag{3.6}$$

In what follows, we will, for a certain model call the set of the three textures defined in eq. (3.6) a "texture set".

Note that in our models, the Z_n flavor symmetries are global, but it might be important to gauge them to survive quantum gravity corrections [35]. The cancellation of anomalies for our symmetries could, e.g. be achieved by considering suitable extra matter fields which is, however, beyond the scope of this paper (for a recent related discussion of anomalies and other phenomenology, see ref. [36]).

4. Scanning approach

In this section, we will describe how we identify among the models introduced in section 3 those which give nearly tribimaximal lepton mixing. First, we pick some flavor symmetry group G_F as in eq. (3.1) and assign to the three generations of leptons e_i^c, ℓ_i, and ν_i^c, all possible charge combinations under G_F according to eqs. (3.2) and (3.3). Then, we determine from the charge assignments the corresponding lepton textures for M_ℓ, M_D, and M_R, following eq. (3.6). Next, to find models that can give a good fit to nearly tribimaximal lepton mixing, we compare the textures found in eq. (3.6) with the list of 1981 representative texture sets given in ref. [31]. In ref. [31], we have, based on assumptions of EQLC, determined fits of the order one Yukawa couplings Y'_ℓ, Y'_D, and Y'_R, to reproduce nearly tribimaximal lepton mixing along with a charged lepton mass spectrum $m_e : m_\mu : m_\tau = \epsilon^4 : \epsilon^2 : 1$ and a normal neutrino mass hierarchy $m_1 : m_2 : m_3 = \epsilon^2 : \epsilon : 1$ in perfect agreement with current neutrino data (at the 3σ confidence level (CL)).

In ref. [31], the textures have been extracted in the basis where $U_{\ell'} = \mathbb{1}$, i.e., where the rotations acting on the e_i^c are zero. Although these rotations do not show up in the observables, they are important for formulating our explicit models using flavor symmetries.

We therefore include now in our considerations textures of the charged leptons that have been extracted in bases where $U_{\ell'}$ can be nontrivial, i.e., where we can have $U_{\ell'} \neq \mathbb{1}$, thereby leading to a multitude of new explicit representations for the charged lepton textures. In complete analogy with ref. [31], we assume for the mixing angles and phases entering the matrices $U_{\ell'}$ all possible combinations that satisfy

$$\theta_{12}^{\ell'}, \theta_{13}^{\ell'}, \theta_{23}^{\ell'} \in \left\{0, \epsilon^2, \epsilon, \frac{\pi}{4}\right\}, \tag{4.1}$$

while the phases can take the values $\widehat{\delta}^{\ell'}, \varphi_1^{\ell'}, \varphi_2^{\ell'} \in \{0, \pi\}$, whereas we can always choose $\varphi_3^{\ell'} = \alpha_1^{\ell'} = \alpha_2^{\ell'} = 0$ (for a definition of the notation, see section 2). This applies the hypothesis of EQLC to the e_i^c such that the left- and right-handed lepton sectors are now all treated on the same footing. As a consequence, after employing the texture reduction from refs. [30, 31], we finally arrive at an enlarged "reference list" of 43278 qualitatively different lepton texture sets. We use this list as our reference to match onto flavor symmetry models for nearly tribimaximal lepton mixing. Any flavor charge assignment that yields textures contained in this reference list provides a valid model that allows an excellent fit to nearly tribimaximal lepton mixing.[1]

We can now impose extra assumptions on the properties of the textures in our reference list to search for interesting flavor symmetry models. For example, we will demand that none of the textures is completely anarchic (or democratic) and that the charged lepton textures contain (after factoring out common factors) at least one entry ϵ^n with $n \geq 4$ (to have sufficient structure in the texture). This reduces the above reference list to a reduced list of 17772 distinct texture sets. In the next section, we will use this reduced reference list to perform the group space scan of flavor symmetries.

5. Results of the group space scan

Let us now present the results of the group space scan for flavor symmetries that produce nearly tribimaximal lepton mixing. We assume throughout the models introduced in section 3 and choose as flavor symmetries[2] $G_F = \Pi_{k=1}^m Z_{n_k}$ for $m = 1, 2, 3, 4$. A complete scan has been performed for groups up to order 40 (for $m = 1$), 45 (for $m = 2$), 30 (for $m = 3$), and 24 (for $m = 4$), with $n_k \leq 9$ for $m > 1$. Valid models are selected as described in section 4 by matching the textures generated by the flavor charge assignments onto the reduced reference list of 17772 non-anarchic texture sets. In total, we find in the scan 6021 such models that reproduce 2093 texture sets. The distribution of the models for the different groups G_F is summarized in figure 1.[3] All these models allow for an excellent fit to nearly tribimaximal neutrino mixing (at 3σ CL and most of them actually at 1σ CL [31])

[1] Note that in our reference list (after factoring out a possible overall power of ϵ) we set an entry ϵ^n equal to zero when $n \geq 3$ (for neutrinos only) or when $n \geq 5$ (for charged leptons). This is different from ref. [31], where such an entry is set to zero when $n \geq 3$ for both neutrinos and charged leptons. For specific flavor models, however, we will always show the actual suppression factor ϵ^n, as predicted by the flavor symmetry.

[2] For an application of related flavor groups see ref. [37].

[3] Additionally, we have also included the number of valid models for $G_F = Z_2 \times Z_4 \times Z_5$.

JHEP04(2008)077

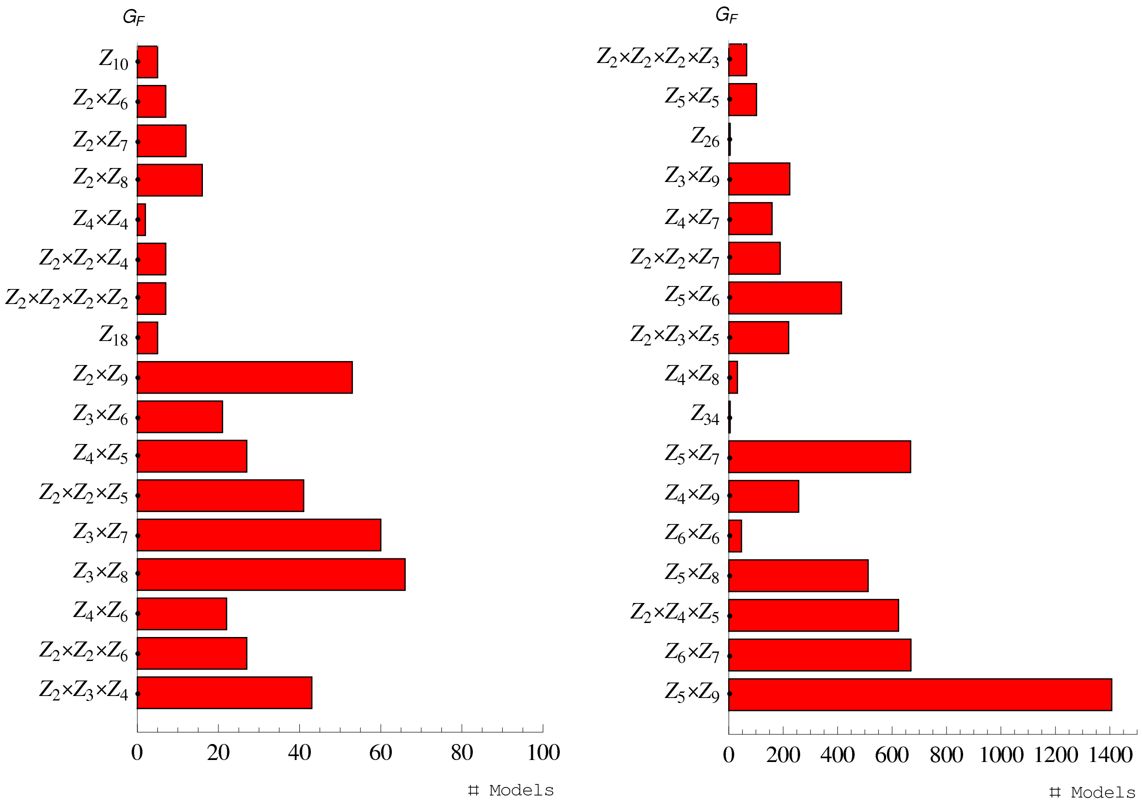

Figure 1: Number of flavor models leading to nearly tribimaximal lepton mixing as a function of the flavor group G_F for increasing group order. In the left (right) panel we have $10 \leq |G_F| \leq 24$ $(24 \leq |G_F| \leq 45)$.

with a very small reactor angle $\theta_{13} \lesssim 1^\circ$ and the lepton mass ratios[4]

$$m_e : m_\mu : m_\tau = \epsilon^4 : \epsilon^2 : 1, \quad m_1 : m_2 : m_3 = \epsilon^2 : \epsilon : 1, \tag{5.1}$$

i.e., we have have a normal neutrino mass hierarchy with a ratio $\Delta m^2_\odot / \Delta m^2_{\text{atm}} \sim \epsilon^2$ of solar over atmospheric neutrino mass squared difference. Since the neutrinos have a normal mass hierarchy, renormalization group effects are negligible (for a discussion and references see ref. [31]).

In figure 1, we can observe the trend that the number of valid models generally increases with the group order $|G_F|$ up to some periodic modulation. It is interesting to give a rough estimate on how large G_F has to be in order to more or less reproduce an arbitrary texture set. For this purpose, note that (after factoring out a possible common overall factor ϵ^n) we restrict ourselves in M_D and M_R to the matrix entries $\{0, \epsilon^2, \epsilon, 1\}$, and in M_ℓ to the entries $\{0, \epsilon^4, \epsilon^3, \epsilon^2, \epsilon, 1\}$, i.e., we have $4^{6+9} \cdot 6^9$ different possibilities for arbitrary texture sets. On the other hand, we have nine different charges per group, leading to $|G_F|^9$ different possibilities for the charge assignments in G_F (cf. eq. (3.2)). Note that we use $|G_F|$ as a figure of merit: The larger $|G_F|$, the more possibilities for the charge assignments we

[4]We are interested here in an SU(5) compatible fit.

have. In order to reproduce *any* texture set, we roughly estimate that the number of possibilities for the charges should exceed the number of possibilities for the texture sets, i.e.,

$$|G_F| \gtrsim 4^{\frac{15}{9}} \cdot 6 \simeq 60. \tag{5.2}$$

This means that, for instance, four Z_n factors with moderate n_k, such as $Z_2 \times Z_3 \times Z_4 \times Z_5$ should be sufficient, or two Z_n factors with large enough n_k, such as Z_7 and higher. In figure 1, we have much less possibilities on the l.h.s. where $10 \leq |G_F| \leq 24$, while we have on the r.h.s. $24 \leq |G_F| \leq 45$. In fact, figure 1 seems to suggest that $G_F = Z_5 \times Z_9$ with $|G_F| = 45$ is already entering the regime estimated in eq. (5.2).

We have checked that for the 6021 valid models practically all (i.e., more than 99%) of the Yukawa coupling matrix elements of Y'_ℓ, Y'_D, and Y'_R, (see eq. (3.4)) lie in the interval between ϵ and $1/\epsilon$. With respect to the expansion parameter ϵ of our models, these matrix elements can therefore indeed be viewed as order one coefficients.

Out of our set of 6021 valid models, let us now consider a few examples. In table 1, we show 22 explicit valid models by listing the flavor group with complete flavor charge assignment and the resulting textures for M_ℓ, M_D, and M_R. The rough guideline for choosing these 22 models was to give one example for each texture set previously identified in EQLC [31], with distinct M_R, a charged lepton mass spectrum as in eq. (5.1), and "natural" order one Yukawa couplings Y'_ℓ, Y'_D, and Y'_R. The complete information on the corresponding mass and mixing parameters of the 22 models is summarized in table 2 in the appendix.

Table 2 demonstrates that our models are very general in the sense that they can exhibit maximal mixings in the charged lepton and/or the neutrino sector, and that the left- and/or right-handed neutrinos can mix maximally. In addition, we can have maximal mixings in all sectors, not only between the 2nd and 3rd generation, but also between the 1st and the 2nd as well as between the 1st and the 3rd generation.

All 22 models in tables 1 and 2 lead to the following PMNS mixing angles in the range

$$34^\circ \lesssim \theta_{12} \lesssim 39^\circ, \quad \theta_{13} \lesssim 1^\circ, \quad \theta_{23} \approx 52^\circ, \tag{5.3}$$

in agreement with neutrino oscillation data (at 3σ CL). The models are thus characterized by a very small reactor angle close to zero, and a significant deviation of about $+7^\circ$ from maximal atmospheric mixing.

Some of the models in table 1 allow to extract very easily new sum rules for the leptonic mixing angles using an expansion in ϵ. For example, we find for model #6 the sum rules

$$\theta_{12} = \frac{\pi}{4} - \frac{\epsilon}{\sqrt{2}} - \frac{\epsilon^2}{4}, \qquad \theta_{13} = \left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right)\epsilon^2, \qquad \theta_{23} = \frac{\pi}{4} + \frac{\epsilon}{\sqrt{2}} + \left(\frac{1}{\sqrt{2}} - \frac{5}{4}\right)\epsilon^2, \tag{5.4}$$

while model #8 exhibits the relations

$$\theta_{12} = \frac{\pi}{4} - \frac{\epsilon}{\sqrt{2}} + \left(\frac{1}{\sqrt{2}} - \frac{9}{4}\right)\epsilon^2, \qquad \theta_{13} = \frac{\epsilon^2}{2}, \qquad \theta_{23} = \frac{\pi}{4} + \frac{\epsilon}{\sqrt{2}} + \left(\frac{1}{\sqrt{2}} - \frac{5}{4}\right)\epsilon^2. \tag{5.5}$$

JHEP04(2008)077

#	$M_\ell/\langle H\rangle$	$M_D/\langle H\rangle$	M_R/M_{B-L}	p^1,p^2,p^3 q^1,q^2,q^3 r^1,r^2,r^3	G_F
1	$\begin{pmatrix}\epsilon^4 & \epsilon^5 & \epsilon^2\\ \epsilon^2 & \epsilon^2 & \epsilon^2\\ \epsilon^2 & \epsilon^4 & 1\end{pmatrix}$	$\epsilon\begin{pmatrix}\epsilon & \epsilon^2 & \epsilon^2\\ \epsilon & 1 & \epsilon\\ \epsilon & 1 & \epsilon\end{pmatrix}$	$\epsilon^3\begin{pmatrix}1 & \epsilon^2 & 1\\ \epsilon^2 & 1 & \epsilon^2\\ 1 & \epsilon^2 & 1\end{pmatrix}$	$(2,0),(0,0),(2,5)$ $(2,3),(4,1),(3,2)$ $(1,4),(2,6),(0,5)$	$Z_5\times Z_7$
2	$\epsilon\begin{pmatrix}\epsilon^4 & \epsilon^4 & \epsilon^2\\ \epsilon^3 & \epsilon^2 & 1\\ \epsilon^3 & \epsilon^4 & 1\end{pmatrix}$	$\epsilon\begin{pmatrix}\epsilon & \epsilon^3 & \epsilon\\ \epsilon & 1 & \epsilon^3\\ \epsilon & \epsilon^2 & \epsilon\end{pmatrix}$	$\epsilon^2\begin{pmatrix}\epsilon & \epsilon & \epsilon\\ \epsilon & 1 & \epsilon^2\\ \epsilon & \epsilon^2 & 1\end{pmatrix}$	$(2,2),(3,2),(2,5)$ $(0,1),(2,2),(4,2)$ $(2,6),(3,4),(1,0)$	$Z_5\times Z_7$
3	$\epsilon\begin{pmatrix}\epsilon^4 & \epsilon^3 & \epsilon^5\\ \epsilon^3 & \epsilon^2 & \epsilon^2\\ \epsilon & \epsilon^2 & 1\end{pmatrix}$	$\epsilon^2\begin{pmatrix}\epsilon & \epsilon & \epsilon^3\\ \epsilon & 1 & 1\\ \epsilon & 1 & 1\end{pmatrix}$	$\epsilon\begin{pmatrix}\epsilon & \epsilon & \epsilon^5\\ \epsilon & 1 & \epsilon^4\\ \epsilon^5 & \epsilon^4 & 1\end{pmatrix}$	$(3,7),(3,0),(2,7)$ $(1,5),(3,6),(3,2)$ $(1,4),(2,4),(2,0)$	$Z_5\times Z_8$
4	$\epsilon\begin{pmatrix}\epsilon^3 & \epsilon^3 & \epsilon\\ \epsilon^2 & \epsilon^2 & \epsilon^2\\ \epsilon^2 & \epsilon^4 & 1\end{pmatrix}$	$\epsilon^3\begin{pmatrix}\epsilon & \epsilon & \epsilon\\ \epsilon & 1 & 1\\ \epsilon & 1 & 1\end{pmatrix}$	$\epsilon\begin{pmatrix}\epsilon & \epsilon^5 & \epsilon\\ \epsilon^5 & 1 & \epsilon^4\\ \epsilon & \epsilon^4 & 1\end{pmatrix}$	$(3,0),(0,1),(2,5)$ $(4,2),(3,6),(3,2)$ $(4,0),(3,4),(3,0)$	$Z_5\times Z_8$
5	$\epsilon\begin{pmatrix}\epsilon^4 & \epsilon^2 & \epsilon\\ \epsilon^3 & \epsilon^2 & \epsilon^2\\ \epsilon^5 & \epsilon & 1\end{pmatrix}$	$\epsilon^2\begin{pmatrix}\epsilon & \epsilon & \epsilon^3\\ \epsilon^3 & 1 & \epsilon\\ \epsilon^3 & 1 & \epsilon\end{pmatrix}$	$\epsilon^2\begin{pmatrix}1 & \epsilon^3 & 1\\ \epsilon^3 & 1 & \epsilon^4\\ 1 & \epsilon^4 & 1\end{pmatrix}$	$(3,8),(4,3),(0,3)$ $(0,4),(3,7),(4,6)$ $(0,8),(2,4),(1,0)$	$Z_5\times Z_9$
6	$\epsilon\begin{pmatrix}\epsilon^4 & \epsilon^2 & \epsilon\\ \epsilon^3 & \epsilon^2 & \epsilon^2\\ \epsilon^5 & \epsilon & 1\end{pmatrix}$	$\epsilon^2\begin{pmatrix}\epsilon^2 & \epsilon^3 & \epsilon\\ \epsilon & 1 & 1\\ \epsilon & 1 & 1\end{pmatrix}$	$\epsilon^2\begin{pmatrix}\epsilon^2 & \epsilon & \epsilon\\ \epsilon & 1 & 1\\ \epsilon & 1 & 1\end{pmatrix}$	$(3,6),(2,1),(1,1)$ $(4,6),(1,0),(0,8)$ $(1,8),(0,8),(1,0)$	$Z_5\times Z_9$
7	$\epsilon\begin{pmatrix}\epsilon^4 & \epsilon^4 & \epsilon^2\\ \epsilon^3 & \epsilon^2 & 1\\ \epsilon^3 & \epsilon^4 & 1\end{pmatrix}$	$\epsilon^4\begin{pmatrix}\epsilon^2 & \epsilon & \epsilon^2\\ \epsilon & \epsilon & \epsilon\\ 1 & \epsilon & 1\end{pmatrix}$	$\epsilon\begin{pmatrix}1 & \epsilon^2 & 1\\ \epsilon^2 & \epsilon & \epsilon^2\\ 1 & \epsilon^2 & 1\end{pmatrix}$	$(2,8),(1,8),(1,4)$ $(1,4),(4,4),(3,5)$ $(2,0),(0,1),(2,0)$	$Z_5\times Z_9$
8	$\epsilon\begin{pmatrix}\epsilon^3 & \epsilon^4 & \epsilon\\ \epsilon^3 & \epsilon^2 & \epsilon^2\\ \epsilon^2 & \epsilon^4 & 1\end{pmatrix}$	$\epsilon^2\begin{pmatrix}\epsilon^2 & \epsilon & \epsilon^3\\ \epsilon^2 & 1 & 1\\ \epsilon^2 & 1 & 1\end{pmatrix}$	$\epsilon^2\begin{pmatrix}\epsilon^2 & \epsilon^2 & \epsilon^2\\ \epsilon^2 & 1 & 1\\ \epsilon^2 & 1 & 1\end{pmatrix}$	$(2,1),(1,6),(4,1)$ $(1,6),(0,1),(1,0)$ $(3,6),(0,1),(1,0)$	$Z_5\times Z_9$
9	$\begin{pmatrix}\epsilon^4 & \epsilon^3 & \epsilon^5\\ \epsilon^3 & \epsilon^2 & \epsilon^2\\ \epsilon & \epsilon^4 & 1\end{pmatrix}$	$\epsilon^3\begin{pmatrix}\epsilon^2 & \epsilon & \epsilon\\ 1 & 1 & \epsilon^2\\ 1 & 1 & \epsilon\end{pmatrix}$	$\epsilon^2\begin{pmatrix}1 & 1 & \epsilon\\ 1 & 1 & \epsilon\\ \epsilon & \epsilon & 1\end{pmatrix}$	$(0,2),(2,5),(1,2)$ $(2,3),(4,4),(5,5)$ $(2,0),(3,1),(0,6)$	$Z_6\times Z_7$
10	$\begin{pmatrix}\epsilon^4 & \epsilon^6 & \epsilon^2\\ \epsilon^2 & \epsilon^2 & \epsilon^2\\ \epsilon^2 & \epsilon^4 & 1\end{pmatrix}$	$\epsilon^2\begin{pmatrix}\epsilon^2 & \epsilon & \epsilon^2\\ \epsilon^2 & \epsilon & 1\\ \epsilon^2 & \epsilon & 1\end{pmatrix}$	$\epsilon\begin{pmatrix}\epsilon^2 & \epsilon^2 & \epsilon^2\\ \epsilon^2 & \epsilon & \epsilon^4\\ \epsilon^2 & \epsilon^4 & 1\end{pmatrix}$	$(2,1),(2,3),(0,1)$ $(1,0),(5,5),(0,6)$ $(4,3),(2,0),(0,3)$	$Z_6\times Z_7$
11	$\begin{pmatrix}\epsilon^3 & \epsilon^3 & \epsilon^2\\ \epsilon^2 & \epsilon^2 & \epsilon^2\\ \epsilon^4 & \epsilon^2 & 1\end{pmatrix}$	$\epsilon^2\begin{pmatrix}\epsilon & \epsilon & \epsilon\\ \epsilon & 1 & \epsilon\\ \epsilon & 1 & \epsilon\end{pmatrix}$	$\epsilon\begin{pmatrix}\epsilon & \epsilon & \epsilon^2\\ \epsilon & 1 & \epsilon^3\\ \epsilon^2 & \epsilon^3 & 1\end{pmatrix}$	$(0,0,0),(1,0,2),(1,1,3)$ $(1,1,1),(0,0,2),(1,2,2)$ $(1,0,1),(1,0,2),(0,1,0)$	$Z_2\times Z_3\times Z_5$

12	$\begin{pmatrix} \epsilon^4 & \epsilon^5 & \epsilon \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^5 & \epsilon^4 & 1 \end{pmatrix}$	$\epsilon^2 \begin{pmatrix} \epsilon & \epsilon & \epsilon^2 \\ \epsilon^2 & 1 & \epsilon^2 \\ \epsilon^2 & 1 & \epsilon \end{pmatrix}$	$\epsilon^2 \begin{pmatrix} 1 & \epsilon & 1 \\ \epsilon & 1 & \epsilon \\ 1 & \epsilon & 1 \end{pmatrix}$	$(1,2,4),(1,1,4),(0,0,2)$ $(0,1,3),(1,0,2),(0,0,3)$ $(0,2,4),(1,0,1),(0,1,0)$	$Z_2 \times Z_4 \times Z_5$
13	$\begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^5 & \epsilon^4 & 1 \end{pmatrix}$	$\epsilon^2 \begin{pmatrix} \epsilon^3 & \epsilon & \epsilon^2 \\ \epsilon^2 & \epsilon & 1 \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon^2 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}$	$(1,1,1),(1,1,0),(0,3,4)$ $(0,2,2),(0,3,1),(0,1,1)$ $(1,0,1),(0,0,2),(0,0,0)$	$Z_2 \times Z_4 \times Z_5$
14	$\epsilon \begin{pmatrix} \epsilon^4 & \epsilon^5 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \\ \epsilon^3 & \epsilon^4 & 1 \end{pmatrix}$	$\epsilon \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon^2 \\ \epsilon^2 & \epsilon & \epsilon^2 \\ 1 & \epsilon^3 & 1 \end{pmatrix}$	$\epsilon^3 \begin{pmatrix} 1 & \epsilon & 1 \\ \epsilon & \epsilon & \epsilon \\ 1 & \epsilon & 1 \end{pmatrix}$	$(0,3),(3,3),(0,6)$ $(3,1),(1,3),(3,3)$ $(1,5),(3,8),(1,5)$	$Z_4 \times Z_9$
15	$\epsilon \begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon^2 \\ \epsilon^3 & \epsilon^4 & 1 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}$	$\epsilon^2 \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon^3 \\ \epsilon^2 & \epsilon & \epsilon^2 \\ \epsilon & \epsilon^2 & 1 \end{pmatrix}$	$\epsilon \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & \epsilon^2 \\ \epsilon & \epsilon^2 & 1 \end{pmatrix}$	$(4,2),(0,2),(4,5)$ $(3,1),(2,2),(0,2)$ $(1,3),(2,3),(0,3)$	$Z_5 \times Z_7$
16	$\epsilon^2 \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \\ \epsilon^3 & \epsilon^4 & 1 \end{pmatrix}$	$\epsilon^2 \begin{pmatrix} \epsilon^3 & \epsilon & \epsilon \\ 1 & 1 & \epsilon^3 \\ \epsilon^2 & \epsilon & \epsilon \end{pmatrix}$	$\epsilon^2 \begin{pmatrix} 1 & 1 & \epsilon^3 \\ 1 & 1 & \epsilon^3 \\ \epsilon^3 & \epsilon^3 & 1 \end{pmatrix}$	$(1,6),(0,5),(1,0)$ $(2,7),(0,8),(3,8)$ $(0,8),(4,0),(2,4)$	$Z_5 \times Z_9$
17	$\epsilon \begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon^2 \\ \epsilon^3 & \epsilon^4 & 1 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}$	$\epsilon \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon^3 \\ \epsilon^2 & \epsilon & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}$	$\epsilon^2 \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon^3 \\ \epsilon^2 & \epsilon & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}$	$(4,3),(0,3),(4,0)$ $(3,1),(2,0),(0,0)$ $(2,2),(2,1),(0,1)$	$Z_5 \times Z_7$
18	$\begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^5 & \epsilon^2 & 1 \end{pmatrix}$	$\epsilon^2 \begin{pmatrix} \epsilon^4 & \epsilon & \epsilon^2 \\ \epsilon^2 & \epsilon & 1 \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}$	$\epsilon \begin{pmatrix} \epsilon^2 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon & \epsilon^5 \\ \epsilon^2 & \epsilon^5 & 1 \end{pmatrix}$	$(4,4),(1,2),(0,1)$ $(1,0),(1,5),(0,6)$ $(2,3),(3,1),(0,3)$	$Z_6 \times Z_7$
19	$\begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}$	$\epsilon^2 \begin{pmatrix} \epsilon & \epsilon^2 & \epsilon \\ \epsilon & 1 & \epsilon \\ \epsilon & 1 & \epsilon \end{pmatrix}$	$\begin{pmatrix} \epsilon & \epsilon^2 & \epsilon^5 \\ \epsilon^2 & 1 & \epsilon^3 \\ \epsilon^5 & \epsilon^3 & 1 \end{pmatrix}$	$(0,1),(0,3),(4,4)$ $(2,1),(1,4),(1,2)$ $(2,3),(0,3),(0,0)$	$Z_5 \times Z_6$
20	$\begin{pmatrix} \epsilon^4 & \epsilon^5 & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \\ \epsilon^4 & \epsilon^4 & 1 \end{pmatrix}$	$\epsilon \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon^2 \\ \epsilon^2 & \epsilon & 1 \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}$	$\epsilon \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & \epsilon & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}$	$(2,0),(3,5),(1,3)$ $(0,4),(3,2),(4,3)$ $(2,1),(0,4),(2,3)$	$Z_5 \times Z_6$
21	$\begin{pmatrix} \epsilon^4 & \epsilon^5 & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \\ \epsilon^4 & \epsilon^4 & 1 \end{pmatrix}$	$\epsilon \begin{pmatrix} \epsilon & \epsilon^2 & \epsilon \\ \epsilon & 1 & \epsilon \\ \epsilon^3 & 1 & \epsilon \end{pmatrix}$	$\epsilon^2 \begin{pmatrix} \epsilon & \epsilon^2 & \epsilon \\ \epsilon^2 & 1 & \epsilon^3 \\ \epsilon & \epsilon^3 & 1 \end{pmatrix}$	$(3,4),(4,4),(1,2)$ $(3,5),(4,2),(4,4)$ $(2,5),(1,3),(1,0)$	$Z_5 \times Z_6$
22	$\begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & \epsilon^3 \\ \epsilon^5 & \epsilon & 1 \end{pmatrix}$	$\epsilon^2 \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon^2 \\ 1 & \epsilon & 1 \\ 1 & \epsilon^3 & 1 \end{pmatrix}$	$\epsilon \begin{pmatrix} 1 & \epsilon^3 & 1 \\ \epsilon^3 & \epsilon & \epsilon^3 \\ 1 & \epsilon^3 & 1 \end{pmatrix}$	$(2,6),(0,0),(0,1)$ $(0,6),(1,1),(0,8)$ $(1,0),(2,5),(1,0)$	$Z_3 \times Z_9$

Table 1: A list of 22 valid flavor models for nearly tribimaximal lepton mixing. Shown are the explicit flavor charges under the flavor symmetry group G_F and the resulting textures. Possible common overall suppression factors have been factored out of the textures.

JHEP04(2008)077

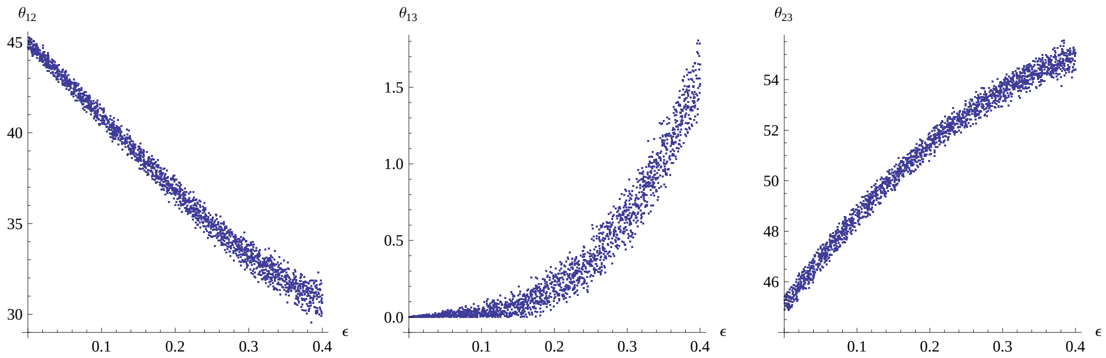

Figure 2: Solar (left), reactor (middle), and atmospheric (right) mixing angles (in degrees), for model #5 in table 1 as a function of the expansion parameter ϵ. The points correspond to a random variation of the order one Yukawa couplings in Y'_ℓ, Y'_D, and Y'_R, by about 1%. The plot in the middle shows that the relation $\theta_{13} \simeq \mathcal{O}(\epsilon^3)$ in eq. (5.6) is not due to an accidental cancellation between large mixing angles.

For both the models #6 and #8, the solar angle satisfies the well-known QLC relation $\theta_{12} \approx \frac{\pi}{4} - \theta_{\rm C}/\sqrt{2}$. The reactor angle, on the other hand, becomes for these two models small due to an apparent suppression by a factor $\sim \theta_{\rm C}^2$. Note that a similar suppression of the reactor angle has been found for the model in ref. [38]. The atmospheric angle follows in both models #6 and #8 the new sum rule $\theta_{23} \approx \frac{\pi}{4} + \theta_{\rm C}/\sqrt{2}$, which predicts a deviation from maximal mixing by an amount of approximately $\theta_{\rm C}/\sqrt{2}$. This prediction makes these models testable in future neutrino oscillation experiments. For example, the deviation from maximal mixing can be established at 3σ CL by the T2K or NOνA experiments [39]. In addition, one can measure the sign of the deviation from maximal mixing (the octant) with a neutrino factory at 3σ CL for $\sin^2 2\theta_{13} \gtrsim 10^{-2.5}$ or at 90% CL otherwise [40].

The relations for θ_{13} in eqs. (5.4) and (5.5), however, are not stable under variations of the Yukawa couplings and must therefore be a result of exact cancellations between large contributions from different sectors. To render these relations for θ_{13} natural, it might, thus, be necessary to extend these models by non-Abelian discrete symmetries as demonstrated in ref. [41].

Let us therefore have a look at model #5, which has the sum rules

$$\theta_{12} = \frac{\pi}{4} - \frac{\epsilon}{\sqrt{2}} - \frac{\epsilon^2}{4}, \qquad \theta_{13} = \mathcal{O}(\epsilon^3), \qquad \theta_{23} = \frac{\pi}{4} + \frac{\epsilon}{\sqrt{2}} - \frac{3}{4}\epsilon^2. \tag{5.6}$$

The sum rules for θ_{12} and θ_{23} are to leading order just as in the two previous examples. The relation for θ_{13}, however, is different from that in eqs. (5.4) and (5.5): Now, θ_{13} is very small due to a suppression by a factor $\sim \theta_{\rm C}^3$. Unlike for models #6 and #8, the relation for θ_{13} in eq. (5.6) is stable under variations of the Yukawa couplings. This means that by varying the order one Yukawa couplings, θ_{13} picks up only a small relative correction that is suppressed by a factor $\sim \theta_{\rm C}^3$. The stability of the relation $\theta_{13} \simeq \mathcal{O}(\epsilon^3)$ under 1% variations of the order one Yukawa couplings is shown in figure 2. While θ_{12} and θ_{23} exhibit variations of the order $(0.5-1)^\circ$ in the limit $\epsilon \to 0$, the variation of θ_{13} remains $\ll 0.1^\circ$. Nevertheless, the sum rules for θ_{12} and θ_{23} are in all three above examples in eqs. (5.4), (5.5), and (5.6), in this sense stable, since the leading term in the respective expansions is $\pi/4$.

In analogy with the Z_n groups, we have also performed a scan of U(1) symmetries. In order to see whether the cyclic character of the discrete groups is a relevant feature in the construction of valid models, we have compared single Z_n groups $G_F = Z_{n_1}$ up to $|G_F| = 40$ and product groups $G_F = Z_{n_1} \times Z_{n_2}$ up to $|G_F| = 18$ with corresponding Abelian groups, i.e., with U(1) (for $G_F = Z_{n_k}$) and U(1) $\times$ U(1) (for $G_F = Z_{n_1} \times Z_{n_2}$) by letting (the absolute value of) the respective individual U(1) charges vary in the whole range $0, 1, \ldots, n_k - 1$. Thereby, we have found 129 models for the cyclic groups and 24 models using U(1) symmetries. All of the 24 U(1) models correspond to the group $G_F = Z_3 \times Z_6$, for which we have obtained in total 21 models. However, even though the U(1) models produce textures that are very similar to those of model #6 in table 1 (with varying M_ℓ and varying first row in M_D), M_D and M_R get in these examples highly suppressed by factors $\lesssim \epsilon^8$. This analysis already indicates that the cyclic character of the flavor groups may be essential for facilitating the construction of realistic flavor models.

6. Summary and conclusions

In this paper, we have constructed several thousand explicit flavor models for nearly tribimaximal lepton mixing from products of Z_n flavor symmetries. In our models, small neutrino masses emerge only from the canonical type-I seesaw mechanism. Upon flavor symmetry breaking, the Froggatt-Nielsen mechanism produces hierarchical Yukawa coupling and mass matrix textures of the leptons. These textures are parameterized by powers of a single small symmetry breaking parameter $\epsilon \simeq \theta_C \simeq 0.2$ that is of the order of the Cabibbo angle θ_C and arises from integrating out heavy Froggatt-Nielsen messenger fermions.

All flavor models realize the assumptions of EQLC and yield an excellent fit to nearly tribimaximal neutrino mixing with a very small reactor angle $\theta_{13} \approx 0$. Moreover, they lead to the hierarchical charged lepton mass spectrum as well as to normal hierarchical neutrino masses. In our analysis, we have restricted ourselves to the most general CP-conserving case of real lepton mass matrices. We have performed a systematic scan of the group space for groups with up to four Z_n factors and a maximum group order of 45. As a consequence, we have found 6021 valid models that reproduce 2093 distinct texture sets.

A characteristic property of our flavor models is that large leptonic mixings can come from the charged leptons and/or neutrinos. In the neutrino sector, maximal mixings can arise in both the Dirac mass matrix or in the heavy right-handed Majorana mass matrix. Generally, we can have maximal mixings between any two generations in any lepton sector.

Among several explicit models, we have found a model that predicts a new relation $\theta_{13} = \mathcal{O}(\theta_C^3)$, for the reactor angle. Moreover, we found models that all satisfy a new sum rule $\theta_{23} \approx \frac{\pi}{4} + \theta_C/\sqrt{2}$ for the atmospheric mixing angle, which makes these models testable in future neutrino oscillation experiments such as the T2K and NOνA experiments or at a neutrino factory.

We wish to point out that in this paper we have established a connection between a model building top-down approach using flavor symmetries and the phenomenological bottom-up approach of ref. [31]. While ref. [31] deals with the extraction of viable lepton mass textures that are in agreement with observation, the current work successfully matches the textures onto explicit flavor models, where the textures are predicted from flavor symmetries and their breaking.

We believe that it would be interesting to study our sample of flavor models with respect to further model building aspects, anomaly cancellation, the inclusion of CP-violating phases, as well as in view of lepton flavor violation or leptogenesis.

A. Mass and mixing parameters

In this appendix, we list the complete information on the mass and mixing parameters of the matrices that are generated by the models in table 1. Here, "#" labels in both tables the same model. The data in table 2 allows to fully reconstruct the exact form of the mass matrices of the 22 models following the notation of section 2 (for further detailed examples on such reconstructions, see also ref. [31]).

#	m_i^D/m_D m_i^R/M_{B-L}	$(\theta_{12}^\ell,\theta_{13}^\ell,\theta_{23}^\ell)$ $(\delta^\ell,\alpha_1^\ell,\alpha_2^\ell)$	$(\theta_{12}^{\ell'},\theta_{13}^{\ell'},\theta_{23}^{\ell'})$ $(\delta^{\ell'},\alpha_1^{\ell'},\alpha_2^{\ell'})$	$(\theta_{12}^D,\theta_{13}^D,\theta_{23}^D)$ $(\delta^D,\varphi_1^D,\varphi_2^D,\varphi_3^D)$	$(\theta_{12}^{D'},\theta_{13}^{D'},\theta_{23}^{D'})$ $(\delta^{D'},\alpha_1^{D'},\alpha_2^{D'})$	$(\theta_{12}^R,\theta_{13}^R,\theta_{23}^R)$ $(\delta^R,\varphi_1^R,\varphi_2^R,\varphi_3^R)$
1	$(\epsilon,1,\epsilon)$ $(\epsilon,1,1)$	$(\epsilon^2,\epsilon^2,\epsilon^2)$ (π,π,π)	$(\frac{\pi}{4},\epsilon^2,0)$ $(0,0,0)$	$(0,\epsilon^2,\frac{\pi}{4})$ $(0,0,0,0)$	$(\epsilon,\epsilon,\epsilon^2)$ $(0,0,0)$	$(\epsilon^2,\frac{\pi}{4},\epsilon^2)$ $(\pi,0,\pi,0)$
2	$(\epsilon,1,\epsilon)$ $(\epsilon,1,1)$	$(\epsilon^2,\epsilon^2,\frac{\pi}{4})$ $(0,0,0)$	$(\epsilon,0,\epsilon^2)$ $(0,0,0)$	$(\epsilon^2,\frac{\pi}{4},\epsilon^2)$ $(0,0,0,0)$	$(\epsilon,\epsilon^2,0)$ $(0,\pi,\pi)$	$(\epsilon,0,\frac{\pi}{4})$ $(0,0,0,0)$
3	$(\epsilon,1,\epsilon)$ $(\epsilon,1,1)$	$(\epsilon,0,\epsilon^2)$ $(0,0,0)$	$(\epsilon,\epsilon,\epsilon^2)$ $(0,0,0)$	$(\epsilon,\frac{\pi}{4},\frac{\pi}{4})$ $(\pi,0,0,\pi)$	$(\epsilon,\epsilon,\frac{\pi}{4})$ $(0,\pi,\pi)$	$(\epsilon,\epsilon,\frac{\pi}{4})$ $(0,0,0,\pi)$
4	$(\epsilon,\epsilon,1)$ $(\epsilon,1,1)$	$(\epsilon,\epsilon,\epsilon^2)$ $(0,\pi,\pi)$	$(\frac{\pi}{4},\epsilon^2,0)$ $(0,0,0)$	$(\frac{\pi}{4},\epsilon^2,\frac{\pi}{4})$ $(0,0,\pi,0)$	$(0,\epsilon^2,\frac{\pi}{4})$ $(0,0,0)$	$(0,\epsilon,\epsilon^2)$ $(0,0,\pi,0)$
5	$(\epsilon,1,\epsilon)$ $(\epsilon,1,1)$	$(0,\epsilon,\epsilon^2)$ $(0,\pi,0)$	$(\epsilon,0,\epsilon)$ $(0,0,0)$	$(\epsilon,\frac{\pi}{4},\frac{\pi}{4})$ $(\pi,0,0,\pi)$	$(\epsilon^2,\frac{\pi}{4},\epsilon^2)$ $(0,\pi,\pi)$	$(0,\frac{\pi}{4},0)$ $(0,0,0,\pi)$
6	$(\epsilon^2,1,\epsilon)$ $(\epsilon^2,\epsilon,1)$	$(0,\epsilon,\epsilon^2)$ $(0,\pi,0)$	$(\epsilon,0,\epsilon)$ $(0,0,0)$	$(\epsilon,\frac{\pi}{4},\frac{\pi}{4})$ $(\pi,0,0,\pi)$	$(\epsilon,\epsilon,\frac{\pi}{4})$ $(\pi,0,0)$	$(\epsilon,\epsilon,\frac{\pi}{4})$ $(\pi,0,\pi,0)$
7	$(\epsilon^2,\epsilon,1)$ $(\epsilon^2,\epsilon,1)$	$(0,\epsilon^2,\frac{\pi}{4})$ $(0,\pi,\pi)$	$(\epsilon,0,\epsilon^2)$ $(0,0,\pi)$	$(\frac{\pi}{4},\epsilon^2,\epsilon)$ $(0,0,0,\pi)$	$(0,\frac{\pi}{4},\epsilon)$ $(0,0,\pi)$	$(\epsilon^2,\frac{\pi}{4},\epsilon^2)$ $(\pi,0,\pi,\pi)$
8	$(\epsilon^2,1,\epsilon)$ $(\epsilon^2,\epsilon,1)$	$(\epsilon^2,\epsilon,\epsilon^2)$ $(0,\pi,0)$	$(\epsilon,\epsilon^2,0)$ $(0,0,0)$	$(\epsilon,\frac{\pi}{4},\frac{\pi}{4})$ $(\pi,0,0,\pi)$	$(0,\epsilon^2,\frac{\pi}{4})$ $(0,0,\pi)$	$(\epsilon^2,\epsilon^2,\frac{\pi}{4})$ $(\pi,0,\pi,0)$
9	$(\epsilon,1,\epsilon)$ $(\epsilon,1,1)$	$(\epsilon,0,\epsilon^2)$ $(0,0,0)$	$(\epsilon,\epsilon,0)$ $(0,0,0)$	$(\epsilon,\frac{\pi}{4},\frac{\pi}{4})$ $(\pi,0,0,\pi)$	$(\frac{\pi}{4},\epsilon,\epsilon^2)$ $(\pi,0,\pi)$	$(\frac{\pi}{4},\epsilon,0)$ $(0,0,0,\pi)$
10	$(\epsilon^2,\epsilon,1)$ $(\epsilon^2,\epsilon,1)$	$(\epsilon^2,\epsilon^2,\epsilon^2)$ (π,π,π)	$(\frac{\pi}{4},\epsilon^2,0)$ $(0,0,0)$	$(\frac{\pi}{4},\epsilon^2,\frac{\pi}{4})$ $(0,0,0,\pi)$	$(\epsilon^2,0,\epsilon^2)$ $(0,0,0)$	$(\epsilon,\epsilon^2,0)$ $(0,0,0,0)$
11	$(\epsilon,1,\epsilon)$ $(\epsilon,1,1)$	$(\epsilon,\epsilon^2,\epsilon^2)$ $(\pi,0,0)$	$(\frac{\pi}{4},0,\epsilon^2)$ $(0,0,0)$	$(\epsilon,\frac{\pi}{4},\frac{\pi}{4})$ $(\pi,0,0,\pi)$	$(\epsilon,\epsilon^2,\epsilon)$ $(\pi,0,0)$	$(\epsilon,\epsilon^2,\epsilon)$ $(\pi,0,0,0)$
12	$(\epsilon,1,\epsilon)$ $(\epsilon,1,1)$	$(0,\epsilon,\epsilon^2)$ $(0,\pi,0)$	$(\epsilon,0,0)$ $(0,0,0)$	$(\epsilon,\frac{\pi}{4},\frac{\pi}{4})$ $(\pi,0,0,\pi)$	$(\epsilon^2,\frac{\pi}{4},\epsilon)$ $(0,\pi,0)$	$(\epsilon,\frac{\pi}{4},\epsilon^2)$ $(\pi,0,\pi,0)$
13	$(\epsilon^2,\epsilon,1)$ $(\epsilon^2,\epsilon,1)$	$(\epsilon^2,\epsilon^2,\epsilon^2)$ $(\pi,0,\pi)$	$(\epsilon,0,0)$ $(0,0,0)$	$(\frac{\pi}{4},\epsilon^2,\frac{\pi}{4})$ $(\pi,0,\pi,0)$	$(\epsilon,\epsilon^2,0)$ $(0,\pi,0)$	$(0,\epsilon^2,\epsilon^2)$ $(0,0,\pi,\pi)$

14	$(\epsilon^2,\epsilon,1)$ $(\epsilon^2,\epsilon,1)$	$(\epsilon^2,\epsilon^2,\frac{\pi}{4})$ (π,π,π)	$(\epsilon,0,\epsilon^2)$ $(0,0,\pi)$	$(\frac{\pi}{4},\epsilon^2,0)$ $(0,0,0,0)$	$(\epsilon^2,\frac{\pi}{4},0)$ $(0,0,\pi)$	$(\epsilon^2,\frac{\pi}{4},\epsilon)$ $(0,0,\pi,0)$
15	$(\epsilon^2,\epsilon,1)$ $(\epsilon^2,\epsilon,1)$	$(\epsilon^2,\epsilon^2,\frac{\pi}{4})$ (π,π,π)	$(\epsilon,0,\epsilon^2)$ $(0,0,0)$	$(\frac{\pi}{4},0,\epsilon^2)$ $(0,0,0,\pi)$	$(\epsilon^2,\epsilon,\epsilon^2)$ $(0,0,0)$	$(\epsilon,\epsilon,\epsilon^2)$ $(\pi,0,0,\pi)$
16	$(\epsilon,1,\epsilon)$ $(\epsilon,1,1)$	$(\epsilon,\epsilon^2,\frac{\pi}{4})$ $(\pi,0,0)$	$(\epsilon,0,\epsilon^2)$ $(0,0,0)$	$(\epsilon,\frac{\pi}{4},\epsilon^2)$ $(\pi,0,0,\pi)$	$(\frac{\pi}{4},0,0)$ $(0,0,\pi)$	$(\frac{\pi}{4},\epsilon^2,\epsilon^2)$ $(0,0,0,0)$
17	$(\epsilon^2,\epsilon,1)$ $(\epsilon^2,\epsilon,1)$	$(\epsilon^2,\epsilon^2,\frac{\pi}{4})$ (π,π,π)	$(\epsilon,0,\epsilon^2)$ $(0,0,0)$	$(\frac{\pi}{4},0,\epsilon^2)$ $(0,0,0,\pi)$	$(\epsilon^2,0,\epsilon^2)$ $(0,0,0)$	$(\epsilon,0,\epsilon^2)$ $(0,0,0,\pi)$
18	$(\epsilon^2,\epsilon,1)$ $(\epsilon^2,\epsilon,1)$	$(\epsilon^2,\epsilon^2,\epsilon^2)$ $(\pi,0,\pi)$	$(\epsilon,0,\epsilon^2)$ $(0,0,0)$	$(\frac{\pi}{4},\epsilon^2,\frac{\pi}{4})$ $(\pi,0,\pi,0)$	$(\epsilon,\epsilon^2,\epsilon^2)$ $(\pi,0,\pi)$	$(0,\epsilon^2,0)$ $(0,0,0,0)$
19	$(\epsilon,1,\epsilon)$ $(\epsilon,1,1)$	$(\epsilon^2,\epsilon^2,\epsilon^2)$ (π,π,π)	$(\frac{\pi}{4},0,\epsilon^2)$ $(0,0,0)$	$(\epsilon^2,\frac{\pi}{4},\frac{\pi}{4})$ $(\pi,0,0,0)$	$(0,\epsilon,\epsilon)$ $(0,0,\pi)$	$(\epsilon^2,\epsilon^2,\frac{\pi}{4})$ $(0,0,0,0)$
20	$(\epsilon^2,\epsilon,1)$ $(\epsilon^2,\epsilon,1)$	$(\epsilon^2,\epsilon^2,\epsilon^2)$ $(0,\pi,\pi)$	$(\frac{\pi}{4},0,0)$ $(0,0,0)$	$(\frac{\pi}{4},\epsilon^2,\frac{\pi}{4})$ $(\pi,0,0,\pi)$	$(\epsilon^2,\epsilon^2,\epsilon^2)$ $(0,0,0)$	$(\epsilon,\epsilon^2,\epsilon^2)$ $(\pi,0,0,\pi)$
21	$(\epsilon,1,\epsilon)$ $(\epsilon,1,1)$	$(\epsilon^2,\epsilon^2,\epsilon^2)$ (π,π,π)	$(\frac{\pi}{4},0,0)$ $(0,0,0)$	$(0,\epsilon^2,\frac{\pi}{4})$ $(0,0,0,0)$	$(\epsilon,\frac{\pi}{4},\epsilon)$ $(0,0,0)$	$(\epsilon^2,\epsilon,\epsilon^2)$ $(\pi,0,\pi,0)$
22	$(\epsilon^2,\epsilon,1)$ $(\epsilon^2,\epsilon,1)$	$(\epsilon^2,\epsilon^2,0)$ $(0,0,0)$	$(\frac{\pi}{4},0,\epsilon)$ $(0,0,0)$	$(\frac{\pi}{4},\epsilon^2,\frac{\pi}{4})$ $(\pi,0,0,0)$	$(\epsilon^2,\frac{\pi}{4},\epsilon)$ $(0,\pi,0)$	$(\epsilon^2,\frac{\pi}{4},0)$ $(0,0,\pi,0)$

Table 2: Supplementary information for the reconstruction of the mass matrices and Yukawa couplings of the models in table 1. Note that we have made use of the freedom to set $\varphi_i^\ell = \varphi_i^{\ell'} = \varphi_i^{D'} = \alpha_j^D = 0$ for $i = 1,2,3$ and $j = 1,2$.

Acknowledgments

The research of F.P. is supported by Research Training Group 1147 *Theoretical Astrophysics and Particle Physics* of Deutsche Forschungsgemeinschaft. G.S. is supported by the Federal Ministry of Education and Research (BMBF) under contract number 05HT1WWA2. W.W. would like to acknowledge support from the Emmy Noether program of Deutsche Forschungsgemeinschaft.

References

[1] Super-Kamiokande collaboration, S. Fukuda et al., *Determination of solar neutrino oscillation parameters using 1496 days of Super-Kamiokande-I data*, *Phys. Lett.* **B 539** (2002) 179 [hep-ex/0205075];
SNO collaboration, Q.R. Ahmad et al., *Measurement of day and night neutrino energy spectra at SNO and constraints on neutrino mixing parameters*, *Phys. Rev. Lett.* **89** (2002) 011302 [nucl-ex/0204009].

[2] Super-Kamiokande collaboration, Y. Fukuda et al., *Evidence for oscillation of atmospheric neutrinos*, *Phys. Rev. Lett.* **81** (1998) 1562 [hep-ex/9807003].

[3] KamLAND collaboration, T. Araki et al., *Measurement of neutrino oscillation with KamLAND: evidence of spectral distortion*, *Phys. Rev. Lett.* **94** (2005) 081801 [hep-ex/0406035];
CHOOZ collaboration, M. Apollonio et al., *Search for neutrino oscillations on a long base-line at the CHOOZ nuclear power station*, *Eur. Phys. J.* **C 27** (2003) 331 [hep-ex/0301017].

[4] K2K collaboration, E. Aliu et al., *Evidence for muon neutrino oscillation in an accelerator-based experiment*, *Phys. Rev. Lett.* **94** (2005) 081802 [hep-ex/0411038].

[5] H. Georgi and S.L. Glashow, *Unity of all elementary particle forces*, *Phys. Rev. Lett.* **32** (1974) 438;
H. Georgi, *Unified gauge theories*, in *Proceedings of Coral Gables 1975, theories and experiments in high energy physics*, New York (1975).

[6] J.C. Pati and A. Salam, *Lepton number as the fourth color*, *Phys. Rev.* **D 10** (1974) 275 [*Erratum ibid.* **D 11** (1975) 703].

[7] P. Minkowski, *$\mu \to e\gamma$ at a rate of one out of 1-billion muon decays?*, *Phys. Lett.* **B 67** (1977) 421;
T. Yanagida, *Horizontal gauge symmetry and masses of neutrinos*, in *Proceedings of the Workshop on the unified theory and Baryon number in the universe*, KEK, Tsukuba (1979);
M. Gell-Mann, P. Ramond and R. Slansky, *Complex spinors and unified theories*, in *Proceedings of the Workshop on Supergravity*, Stony Brook, New York, (1979);
S.L. Glashow, *The future of elementary particle physics*, in *Proceedings of the 1979 Cargese Summer Institute on Quarks and Leptons*, New York (1980).

[8] M. Magg and C. Wetterich, *Neutrino mass problem and gauge hierarchy*, *Phys. Lett.* **B 94** (1980) 61;
R.N. Mohapatra and G. Senjanović, *Neutrino mass and spontaneous parity nonconservation*, *Phys. Rev. Lett.* **44** (1980) 912; *Neutrino masses and mixings in gauge models with spontaneous parity violation*, *Phys. Rev.* **D 23** (1981) 165;
J. Schechter and J.W.F. Valle, *Neutrino masses in* $\mathrm{SU}(2)\times\mathrm{U}(1)$ *theories*, *Phys. Rev.* **D 22** (1980) 2227;
G. Lazarides, Q. Shafi and C. Wetterich, *Proton lifetime and fermion masses in an* SO(10) *model*, *Nucl. Phys.* **B 181** (1981) 287.

[9] H. Georgi, H.R. Quinn and S. Weinberg, *Hierarchy of interactions in unified gauge theories*, *Phys. Rev. Lett.* **33** (1974) 451;
S. Dimopoulos, S. Raby and F. Wilczek, *Supersymmetry and the scale of unification*, *Phys. Rev.* **D 24** (1981) 1681;
S. Dimopoulos and H. Georgi, *Softly broken supersymmetry and* SU(5), *Nucl. Phys.* **B 193** (1981) 150.

[10] T. Schwetz, *Global fits to neutrino oscillation data*, *Phys. Scripta* **T127** (2006) 1 [hep-ph/0606060].

[11] B. Pontecorvo, *Mesonium and antimesonium*, *Sov. Phys. JETP* **6** (1957) 429;
Z. Maki, M. Nakagawa and S. Sakata, *Remarks on the unified model of elementary particles*, *Prog. Theor. Phys.* **28** (1962) 870.

[12] P.F. Harrison, D.H. Perkins and W.G. Scott, *A redetermination of the neutrino mass-squared difference in tri-maximal mixing with terrestrial matter effects*, *Phys. Lett.* **B 458** (1999) 79

JHEP04(2008)077

[hep-ph/9904297]; *Tri-bimaximal mixing and the neutrino oscillation data*, *Phys. Lett.* **B 530** (2002) 167 [hep-ph/0202074].

[13] F. Plentinger and W. Rodejohann, *Deviations from tribimaximal neutrino mixing*, *Phys. Lett.* **B 625** (2005) 264 [hep-ph/0507143].

[14] D. Majumdar and A. Ghosal, *Probing deviations from tri-bimaximal mixing through ultra high energy neutrino signals*, *Phys. Rev.* **D 75** (2007) 113004 [hep-ph/0608334];
A.H. Chan, H. Fritzsch, S. Luo and Z.-z. Xing, *Deviations from tri-bimaximal neutrino mixing in type-II seesaw and leptogenesis*, *Phys. Rev.* **D 76** (2007) 073009 [arXiv:0704.3153];
S.F. King, *Parametrizing the lepton mixing matrix in terms of deviations from tri-bimaximal mixing*, *Phys. Lett.* **B 659** (2008) 244 [arXiv:0710.0530].

[15] Z.-z. Xing, *Nearly tri-bimaximal neutrino mixing and CP-violation*, *Phys. Lett.* **B 533** (2002) 85 [hep-ph/0204049];
Z.-z. Xing, H. Zhang and S. Zhou, *Nearly tri-bimaximal neutrino mixing and CP-violation from $\mu - \tau$ symmetry breaking*, *Phys. Lett.* **B 641** (2006) 189 [hep-ph/0607091];
E. Ma, *Near tribimaximal neutrino mixing with Δ_{27} symmetry*, *Phys. Lett.* **B 660** (2008) 505 [arXiv:0709.0507];
A. Mondragon, M. Mondragon and E. Peinado, *Nearly tri-bimaximal mixing in the S_3 flavour symmetry*, arXiv:0712.2488.

[16] E. Ma and G. Rajasekaran, *Softly broken A_4 symmetry for nearly degenerate neutrino masses*, *Phys. Rev.* **D 64** (2001) 113012 [hep-ph/0106291];
K.S. Babu, E. Ma and J.W.F. Valle, *Underlying A_4 symmetry for the neutrino mass matrix and the quark mixing matrix*, *Phys. Lett.* **B 552** (2003) 207 [hep-ph/0206292];
M. Hirsch, J.C. Romao, S. Skadhauge, J.W.F. Valle and A. Villanova del Moral, *Phenomenological tests of supersymmetric A_4 family symmetry model of neutrino mass*, *Phys. Rev.* **D 69** (2004) 093006 [hep-ph/0312265].

[17] P.H. Frampton and T.W. Kephart, *Simple nonabelian finite flavor groups and fermion masses*, *Int. J. Mod. Phys.* **A 10** (1995) 4689 [hep-ph/9409330];
A. Aranda, C.D. Carone and R.F. Lebed, *Maximal neutrino mixing from a minimal flavor symmetry*, *Phys. Rev.* **D 62** (2000) 016009 [hep-ph/0002044];
P.D. Carr and P.H. Frampton, *Group theoretic bases for tribimaximal mixing*, hep-ph/0701034;
A. Aranda, *Neutrino mixing from the double tetrahedral group T'*, *Phys. Rev.* **D 76** (2007) 111301 [arXiv:0707.3661].

[18] G. Altarelli, *Models of neutrino masses and mixings: a progress report*, arXiv:0705.0860.

[19] N. Cabibbo, *Unitary symmetry and leptonic decays*, *Phys. Rev. Lett.* **10** (1963) 531;
M. Kobayashi and T. Maskawa, *CP violation in the renormalizable theory of weak interaction*, *Prog. Theor. Phys.* **49** (1973) 652.

[20] E. Ma, *Quark mass matrices in the A_4 model*, *Mod. Phys. Lett.* **A 17** (2002) 627 [hep-ph/0203238];
G. Altarelli and F. Feruglio, *Tri-bimaximal neutrino mixing, A_4 and the modular symmetry*, *Nucl. Phys.* **B 741** (2006) 215 [hep-ph/0512103];
C. Hagedorn, M. Lindner and F. Plentinger, *The discrete flavor symmetry D(5)*, *Phys. Rev.* **D 74** (2006) 025007 [hep-ph/0604265];
S.F. King and M. Malinsky, *Towards a complete theory of fermion masses and mixings with* SO(3) *family symmetry and* $5D$ SO(10) *unification*, *JHEP* **11** (2006) 071 [hep-ph/0608021];

A_4 *family symmetry and quark-lepton unification*, *Phys. Lett.* **B 645** (2007) 351 [hep-ph/0610250];
F. Feruglio, C. Hagedorn, Y. Lin and L. Merlo, *Tri-bimaximal neutrino mixing and quark masses from a discrete flavour symmetry*, *Nucl. Phys.* **B 775** (2007) 120 [hep-ph/0702194];
C. Luhn, S. Nasri and P. Ramond, *Tri-bimaximal neutrino mixing and the family symmetry* $Z_7 \rtimes Z_3$, *Phys. Lett.* **B 652** (2007) 27 [arXiv:0706.2341].

[21] E. Ma, H. Sawanaka and M. Tanimoto, *Quark masses and mixing with* A_4 *family symmetry*, *Phys. Lett.* **B 641** (2006) 301 [hep-ph/0606103];
I. de Medeiros Varzielas, S.F. King and G.G. Ross, *Neutrino tri-bi-maximal mixing from a non-Abelian discrete family symmetry*, *Phys. Lett.* **B 648** (2007) 201 [hep-ph/0607045];
E. Ma, *Suitability of* A_4 *as a family symmetry in grand unification*, *Mod. Phys. Lett.* **A 21** (2006) 2931 [hep-ph/0607190];
S. Morisi, M. Picariello and E. Torrente-Lujan, *A model for fermion masses and lepton mixing in* $\mathrm{SO}(10) \times A_4$, *Phys. Rev.* **D 75** (2007) 075015 [hep-ph/0702034];
M.-C. Chen and K.T. Mahanthappa, *CKM and tri-bimaximal MNS matrices in a* $\mathrm{SU}(5) \times^{(d)} T$ *model*, *Phys. Lett.* **B 652** (2007) 34 [arXiv:0705.0714];
W. Grimus and H. Kuhbock, *Embedding the Zee-Wolfenstein neutrino mass matrix in an* $\mathrm{SO}(10) \times A_4$ *GUT scenario*, *Phys. Rev.* **D 77** (2008) 055008 [arXiv:0710.1585];
G. Altarelli, F. Feruglio and C. Hagedorn, *A SUSY* SU(5) *grand unified model of tri-bimaximal mixing from* A_4, arXiv:0802.0090.

[22] A.Y. Smirnov, *Neutrinos: '... Annus mirabilis'*, hep-ph/0402264;
M. Raidal, *Prediction* $\Theta_C + \Theta_{sol} = \pi/4$ *from flavor physics: a new evidence for grand unification?*, *Phys. Rev. Lett.* **93** (2004) 161801 [hep-ph/0404046];
H. Minakata and A.Y. Smirnov, *Neutrino mixing and quark lepton complementarity*, *Phys. Rev.* **D 70** (2004) 073009 [hep-ph/0405088].

[23] M. Jezabek and Y. Sumino, *Neutrino masses and bimaximal mixing*, *Phys. Lett.* **B 457** (1999) 139 [hep-ph/9904382];
C. Giunti and M. Tanimoto, *CP violation in bilarge lepton mixing*, *Phys. Rev.* **D 66** (2002) 113006 [hep-ph/0209169];
P.H. Frampton, S.T. Petcov and W. Rodejohann, *On deviations from bimaximal neutrino mixing*, *Nucl. Phys.* **B 687** (2004) 31 [hep-ph/0401206].

[24] T. Ohlsson, *Bimaximal fermion mixing from the quark and leptonic mixing matrices*, *Phys. Lett.* **B 622** (2005) 159 [hep-ph/0506094];
S. Antusch and S.F. King, *Charged lepton corrections to neutrino mixing angles and CP phases revisited*, *Phys. Lett.* **B 631** (2005) 42 [hep-ph/0508044].

[25] K. Cheung, S.K. Kang, C.S. Kim and J. Lee, *Lepton flavor violation as a probe of quark-lepton unification*, *Phys. Rev.* **D 72** (2005) 036003 [hep-ph/0503122];
K.A. Hochmuth and W. Rodejohann, *Low and high energy phenomenology of quark-lepton complementarity scenarios*, *Phys. Rev.* **D 75** (2007) 073001 [hep-ph/0607103].

[26] W. Rodejohann, *A parametrization for the neutrino mixing matrix*, *Phys. Rev.* **D 69** (2004) 033005 [hep-ph/0309249];
N. Li and B.-Q. Ma, *Unified parametrization of quark and lepton mixing matrices*, *Phys. Rev.* **D 71** (2005) 097301 [hep-ph/0501226];
Z.-z. Xing, *Nontrivial correlation between the CKM and MNS matrices*, *Phys. Lett.* **B 618** (2005) 141 [hep-ph/0503200];

A. Datta, L. Everett and P. Ramond, *Cabibbo haze in lepton mixing*, *Phys. Lett.* **B 620** (2005) 42 [hep-ph/0503222];
L.L. Everett, *Viewing lepton mixing through the Cabibbo haze*, *Phys. Rev.* **D 73** (2006) 013011 [hep-ph/0510256].

[27] B.C. Chauhan, M. Picariello, J. Pulido and E. Torrente-Lujan, *Quark-lepton complementarity, neutrino and standard model data predict* ($\theta_{13}^{PMNS} = 9^{+1}_{-2})^\circ$, *Eur. Phys. J.* **C 50** (2007) 573 [hep-ph/0605032].

[28] A. Dighe, S. Goswami and P. Roy, *Quark-lepton complementarity with quasidegenerate Majorana neutrinos*, *Phys. Rev.* **D 73** (2006) 071301 [hep-ph/0602062];
M.A. Schmidt and A.Y. Smirnov, *Quark lepton complementarity and renormalization group effects*, *Phys. Rev.* **D 74** (2006) 113003 [hep-ph/0607232].

[29] T. Ohlsson and G. Seidl, *A flavor symmetry model for bilarge leptonic mixing and the lepton masses*, *Nucl. Phys.* **B 643** (2002) 247 [hep-ph/0206087];
P.H. Frampton and R.N. Mohapatra, *Possible gauge theoretic origin for quark-lepton complementarity*, *JHEP* **01** (2005) 025 [hep-ph/0407139];
S. Antusch, S.F. King and R.N. Mohapatra, *Quark lepton complementarity in unified theories*, *Phys. Lett.* **B 618** (2005) 150 [hep-ph/0504007];
M. Picariello, *Neutrino CP-violating parameters from nontrivial quark- lepton correlation: a* S^3 *x GUT model*, hep-ph/0611189;
A. Hernandez-Galeana, *Fermion masses and Neutrino mixing in an* $\mathrm{U}(1)_H$ *flavor symmetry model with hierarchical radiative generation for light charged fermion masses*, *Phys. Rev.* **D 76** (2007) 093006 [arXiv:0710.2834].

[30] F. Plentinger, G. Seidl and W. Winter, *Systematic parameter space search of extended quark-lepton complementarity*, *Nucl. Phys.* **B 791** (2008) 60 [hep-ph/0612169].

[31] F. Plentinger, G. Seidl and W. Winter, *The seesaw mechanism in quark-lepton complementarity*, *Phys. Rev.* **D 76** (2007) 113003 [arXiv:0707.2379].

[32] W. Winter, *Neutrino oscillation observables from mass matrix structure*, *Phys. Lett.* **B 659** (2008) 275 [arXiv:0709.2163].

[33] G.C. Branco, D. Emmanuel-Costa, R. Gonzalez Felipe and H. Serodio, *Weak basis transformations and texture zeros in the leptonic sector*, arXiv:0711.1613;
G.C. Branco, D. Emmanuel-Costa, M.N. Rebelo and P. Roy, *Four zero neutrino Yukawa textures in the minimal seesaw framework*, *Phys. Rev.* **D 77** (2008) 053011 [arXiv:0712.0774].

[34] C.D. Froggatt and H.B. Nielsen, *Hierarchy of quark masses, Cabibbo angles and CP-violation*, *Nucl. Phys.* **B 147** (1979) 277.

[35] L.M. Krauss and F. Wilczek, *Discrete Gauge symmetry in continuum theories*, *Phys. Rev. Lett.* **62** (1989) 1221.

[36] H.K. Dreiner, H. Murayama and M. Thormeier, *Anomalous flavor* $\mathrm{U}(1)X$ *for everything*, *Nucl. Phys.* **B 729** (2005) 278 [hep-ph/0312012];
H.K. Dreiner, C. Luhn, H. Murayama and M. Thormeier, *Baryon triality and neutrino masses from an anomalous flavor* $\mathrm{U}(1)$, *Nucl. Phys.* **B 774** (2007) 127 [hep-ph/0610026];
Proton hexality from an anomalous flavor $\mathrm{U}(1)$ *and neutrino masses - linking to the string scale*, *Nucl. Phys.* **B 795** (2008) 172 [arXiv:0708.0989].

[37] J. Sayre and S. Wiesenfeldt, *Naturalness and the neutrino matrix*, *Phys. Rev.* **D 77** (2008) 053005 [arXiv:0711.1687].

[38] F. Feruglio and Y. Lin, *Fermion mass hierarchies and flavour mixing from a minimal discrete symmetry*, arXiv:0712.1528.

[39] S. Antusch, P. Huber, J. Kersten, T. Schwetz and W. Winter, *Is there maximal mixing in the lepton sector?*, *Phys. Rev.* **D 70** (2004) 097302 [hep-ph/0404268].

[40] R. Gandhi and W. Winter, *Physics with a very long neutrino factory baseline*, *Phys. Rev.* **D 75** (2007) 053002 [hep-ph/0612158].

[41] F. Plentinger and G. Seidl, *Mapping out* SU(5) *GUTs with non-Abelian discrete flavor symmetries*, arXiv:0803.2889.